AP* Calculus Review

Related Rates

Student Packet

Related Rates

Session Notes

Questions that ask for the calculation of the rate at which one variable changes, based on the rate at which another variable is known to change, are usually called related rates. Solutions are found by writing an equation that relates the variables of the problem then differentiating them with respect to another variable (usually time). Since time is rarely a variable in the equation that you write, you will have to differentiate implicitly with respect to time.

You can recognize that the rates are with respect to time by looking at the units: $\frac{\text{ft}}{\text{sec}}$, $\frac{\text{cm}^3}{\text{sec}}$, or $\frac{\text{mi}}{\text{hr}}$.

Process

1. Draw a picture. Label constant values and assign variables to things that change.
2. Translate the given information in the problem into "calculus-speak". Do the same thing for what you are asked to find. For example, the rate of change of the area is 15 $\frac{\text{ft}^2}{\text{min}}$ becomes $\frac{dA}{dt} = 15\ \frac{\text{ft}^2}{\text{min}}$.
3. Write a formula/equation relating the variables whose rates of change you seek and the variables whose rates of change you are given. Look to geometry for many formulas.
   Important: At this stage you may substitute for a quantity that is constant; however, don't freeze your problem by substituting a number for a quantity that is changing – keep variables variable!!
4. Differentiate implicitly with respect to time. Use all differentiation rules that apply.
5. Now you can plug in numbers and do calculations. If you round off an answer, use three decimal places.
6. Use complete sentences to answer the question that is asked.

Practice for preliminary steps

Translate into "calculus-speak".

1. The area of a circle is increasing at a rate of $6\ \dfrac{\text{in}^2}{\text{min}}$.

2. The volume of a cone is decreasing at a rate of $2\ \dfrac{\text{ft}^3}{\text{sec}}$.

3. The population of a city is growing at a rate of $3\ \dfrac{\text{people}}{\text{day}}$.

Translate into words.

1. $\dfrac{dC}{dt} = 2\ \dfrac{\text{in}}{\text{sec}}$

2. $\dfrac{dr}{dt} = 40\ \dfrac{\text{cm}}{\text{sec}}$

3. $\dfrac{dV}{dt} = -28\ \dfrac{\text{ft}^3}{\text{min}}$

Differentiate with respect to *t*.

1. $A = \pi r^2$

2. $a^2 + b^2 = c^2$

3. $P = 2L + 2W$

4. $V = \pi r^2 h$

5. $A = \frac{1}{2}bh$

1. The radius of a circle is increasing at a constant rate of 0.4 meters per second. What is the rate of increase in the area of the circle at the instant when the circumference is 60π?

(A) $0.24\ \frac{m^2}{\sec}$ (B) $24\ \frac{m^2}{\sec}$ (C) $24\pi\ \frac{m^2}{\sec}$

(D) $30\pi\ \frac{m^2}{\sec}$ (E) $240\pi\ \frac{m^2}{\sec}$

2. A beach ball is deflating at a constant rate of 10 cubic centimeters per second. When the volume of the ball is $\frac{256}{3}\pi$ cubic centimeters, what is the rate of change of the surface area? ($S = 4\pi r^2$ and $V = \frac{4}{3}\pi r^3$)

(A) $-80\pi \frac{\text{cm}^2}{\sec}$ (B) $-5\frac{\text{cm}^2}{\sec}$ (C) $-\frac{5}{2\pi}\ \frac{cm^2}{\sec}$

(D) $5\ \frac{cm^2}{\sec}$ (E) $320\ \pi\ \frac{cm^2}{\sec}$

3. The height of grain in a cylindrical silo is increasing at a constant rate of 4 feet per minute. At what rate is the volume of grain in the cylinder increasing if the radius of the silo is 10 feet? $(V = \pi r^2 h)$

(A) $\frac{1}{25\pi}\ \frac{ft^3}{\min}$ (B) $25\pi\ \frac{ft^3}{\min}$ (C) $40\pi\ \frac{ft^3}{\min}$

(D) $400\ \frac{ft^3}{\min}$ (E) $400\pi\ \frac{ft^3}{\min}$

4. A 15 foot ladder is leaning against a building when its base begins to slide away from the building at constant rate of 2 feet per minute. How fast is the height of the ladder moving down from the building when the base of the ladder is 9 feet from the building?

(A) $-\frac{64}{9}\ \frac{ft}{\min}$ (B) $\frac{2}{3}\ \frac{ft}{\min}$ (C) $\frac{3}{2}\ \frac{ft}{\min}$

(D) $\frac{8}{3}\ \frac{ft}{\min}$ (E) $\frac{7}{2}\ \frac{ft}{\min}$

5. Bikes A and B are traveling on perpendicular roads. At the same time, bike A is leaving the intersection at a rate of 2 feet per second and bike B is leaving the intersection at 3 feet per second. How fast is the distance, in feet per second, between them changing after 5 seconds?

(A) $-\frac{13}{5}$ (B) $\frac{13}{5}$ (C) $\sqrt{13}$

(D) $\frac{13\sqrt{5}}{5}$ (E) $5\sqrt{13}$

6GC. In a rectangle, the length is increasing at constant rate of 3.02 centimeters per second, while the width is decreasing at a constant rate of 0.62 centimeters per second. At the time that the length is 2 centimeters and the width is 0.4, the rate of change of the area is

(A) -0.032 (B) -1.8724 (C) 1.8724

(D) 2.448 (E) 5.792

7GC. The volume of a spherical balloon ($V = \frac{4}{3}\pi r^3$) is increasing at a constant rate of 0.78 inches per minute. At the instant when the radius is 3.20 inches, the radius is increasing at a rate of

(A) $0.006 \frac{in}{\min}$ (B) $0.019 \frac{in}{\min}$ (C) $0.419 \frac{in}{\min}$

(D) $6.273 \frac{in}{\min}$ (E) $100.37 \frac{in}{\min}$

8.GC On a construction site, gravel is delivered and poured into a conical pile. The diameter and height of the cone of gravel are changing in a way that the diameter is always 3 times the height. If the delivery truck is set to pour the gravel at a constant rate of 3 cubic feet per minute, how fast is the radius of the pile changing when the height is 4 feet? ($V = \frac{1}{3}\pi r^2 h$)

(A) $0.040 \frac{ft}{\min}$ (B) $0.060 \frac{ft}{\min}$ (C) $0.090 \frac{ft}{\min}$

(D) $0.598 \frac{ft}{\min}$ (E) $150.796 \frac{ft}{\min}$

9. A camera is filming the progress as a daredevil attempts to scale the wall of a skyscraper. The climber is moving vertically at a constant rate of 16 feet per minute, and the camera is 400 feet from the base of the skyscraper. Through how many radians per minute is the camera angle changing when the climber is 300 feet up the building?

(A) $\frac{1}{625}$ (B) $\frac{1}{320}$ (C) $\frac{16}{625}$

(D) $\frac{1}{20}$ (E) $\frac{3}{5}$

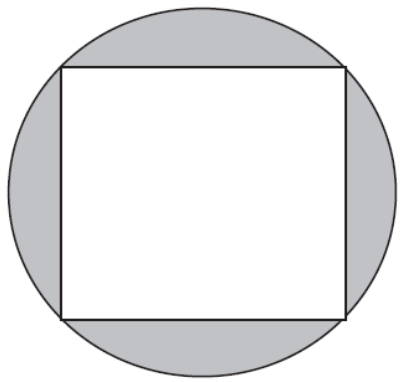
10. A cube has an edge of 40 feet at $t = 0$, and the edge is decreasing at a constant rate of 4 feet per minute. After 2 minutes, the rate of change of the volume in cubic feet per minute is

(A) 384 (B) 480 (C) 6400

(D) 12,288 (E) 19,200

Free Response 1 – No calculator

A square is inscribed in a circle. The radius of the circle is increasing at a constant rate of 0.8 centimeters per second.



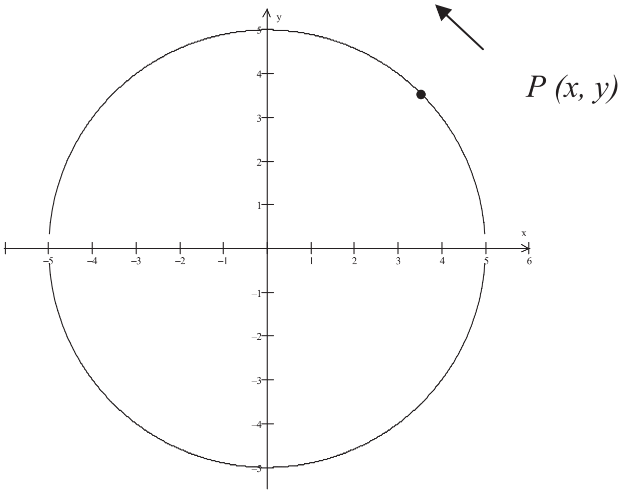
(a) When the side of the square is 4 centimeters, what is the area of the circle? Include units.

(b) When the side of the square is 4 centimeters, what is the rate of change in the area of the circle? Include units.

(c) When the radius of the circle is $5\sqrt{2}$ centimeters, what is the rate of change in the shaded area of the region outside the square but inside the circle? Include units. (at the this time the change of rate of the side of the square is $0.8\sqrt{2}\ cm/\sec$)

Free Response 2 – No calculator

A point, P, with coordinates (x, y) is moving counterclockwise on a circle centered at (0, 0) with radius 5.



(a) When P is at (4, 3) $\frac{dx}{dt}$ is –2. What is $\frac{dy}{dt}$?

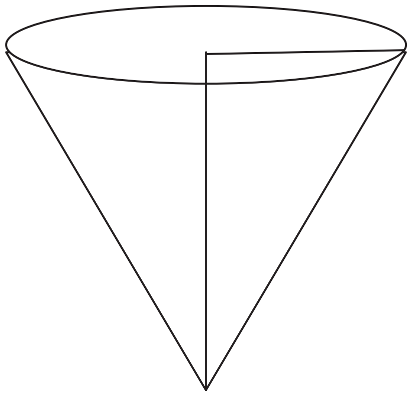
(b) What are the signs of $\frac{dx}{dt}$ and $\frac{dy}{dt}$ in quadrant 2? Justify your answer.

(c) Consider the triangle with vertices (0, 0), $(x, 0)$ and (x, y). Using information from part (a), find the rate at which the area of this triangle is changing when P is at (4, 3).

(d) In the triangle from part (c), θ is the angle formed by the hypotenuse and the horizontal leg. When P is at (4, 3), find the rate of change of θ.

Free Response 3 – Calculator Allowed

Water is poured into a conical tank that is 24 feet tall and has a diameter at the top of 20 feet. ($V = \frac{1}{3}\pi r^2 h$)



(a) Write the formula for the volume of the cone of water in terms of *h*, the height of the water in the tank.

(b) When the volume of the water is increasing at 3.4 cubic feet per minute and the height of the water is 2 feet, at what rate is the height of the water changing?

(c) The radius of the surface of the water in the tank is increasing at 0.75 feet per minute. At what rate is the area of the surface changing when the radius is 4.2 feet?